$$J_x = K_9\sigma B_0 \exp\left[\left(\frac{\sigma B_0^2}{\mu}\right)^{1/2} z\right] + K_{10}\sigma B_0 \times \exp\left[-\left(\frac{\sigma B_0^2}{\mu}\right)^{1/2} z\right] + K_5\left(\frac{1}{B_0}\right)$$

$$J_y = -K_7\sigma B_0 \exp\left[\left(\frac{\sigma B_0^2}{\mu}\right)^{1/2} z\right] - K_8\sigma B_0 \times \exp\left[-\left(\frac{\sigma B_0^2}{\mu}\right)^{1/2} z\right] - K_4\left(\frac{1}{B_0}\right)$$

and these equations can be integrated in accordance with Ampere's law to give the magnetic field as

$$B_x = -K_7(\mu\mu_0^2\sigma)^{1/2} \exp\left[\left(\frac{\sigma B_0^2}{\mu}\right)^{1/2} z\right] - K_8(\mu\mu_0^2\sigma)^{1/2} \exp\left[-\left(\frac{\sigma B_0^2}{\mu}\right)^{1/2} z\right] - K_4\left(\frac{\mu_0}{B_0}\right) z + K_{11}$$

$$B_y = -K_9(\mu\mu_0^2\sigma)^{1/2} \exp\left[\left(\frac{\sigma B_0^2}{\mu}\right)^{1/2} z\right] - K_{10}(\mu\mu_0^2\sigma)^{1/2} \exp\left[-\left(\frac{\sigma B_0^2}{\mu}\right)^{1/2} z\right] - K_5\left(\frac{\mu_0}{B_0}\right) z + K_{12}$$

The solutions just determined show that the correct description of case 2 flows is, therefore,

$$\begin{aligned} \mathbf{V} &= \mathbf{i}u(z) + \mathbf{j}v(z) \\ \mathbf{B} &= \mathbf{i}B_x(z) + \mathbf{j}B_y(z) + \mathbf{k}B_0 \\ P &= P_1(x) + P_2(y) + P_3(z) \\ \mathbf{J} &= \mathbf{i}J_x(z) + \mathbf{j}J_y(z) \\ \mathbf{E} &= \text{const} \end{aligned}$$

In conclusion, the problem encountered in attempting to generalize magnetohydrodynamic flow problems lies in the fact that the governing equations are a set of coupled equations Thus, any restrictions imposed on one variable have effects on the allowable forms for all other variables, and a careful examination of all of the implicit restrictions is essential As has been previously shown, what appears to be a relatively general solution may in fact, turn out to be a rather specialized result

Reference

[1] El-Saden, M R, 'A class of linear magnetohydrodynamic flows,' AIAA J 1, 236–238 (1963)

Reply by Author to T P Anderson

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THE primary purpose of my note[1] was to point out, in a general way, the manner in which the nonlinear terms in the governing equations for incompressible magnetohydrodynamic flow problems may be identically satisfied The boundary conditions outlined in my note do just that If these boundary conditions entail additional restrictions as Anderson has shown, so be it This is beside the primary purpose of my note

Solutions to the linearized equations are available in several publications, some of which were referred to in the note

Reference

[1] El-Saden, M R, 'A class of linear magnetohydrodynamic flows," AIAA J 1, 236–238 (1963)

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Comments on "Numerical Analysis of Unsymmetrical Bending of Shells of Revolution"

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THE paper[1] presented by Budiansky and Radkowski has been of interest to the author in connection with a digital program, similar to that of Ref 1, developed at Space Technology Laboratories for static and dynamic analyses of unsymmetrically loaded shells of revolution However, the conditions to be applied at the pole of a surface of revolution which were given in Ref 1 differ from the ones used in the Space Technology Laboratories program, and it is on this aspect of the paper that the author wishes to comment

In Ref 1, it was stated that a simple-minded way to handle the singularity problem that arises at a pole of a surface of revolution is to choose the boundary $S = 0$, not at the pole, but at a very short distance away, and then to impose boundary conditions at $S = 0$ as

$$\left.\begin{aligned} u_\xi = u_\theta = \phi_\xi = \hat{f}_\xi &= 0 \qquad \text{for } n = 0 \\ t_\xi = \hat{t}_{\xi\theta} = w = m_\xi &= 0 \qquad \text{for } n = 1 \\ u_\xi = u_\theta = w = m_\xi &= 0 \qquad \text{for } n \geqslant 2 \end{aligned}\right\} \quad (1)$$

where n is the Fourier index

The conditions just given for $n = 0$ are correct; however, it will be shown that two of the conditions given for $n = 1$ are not independent, and for this reason an additional condition is required; it will also be shown that, for $n = 2$, the condition $m_\xi = 0$ is incorrect Furthermore, it will be shown that the correct conditions can be applied at the pole so that the point $S = 0$ will be chosen to be at the pole

The conditions to be applied at the pole can be determined by examining the strain-displacement and curvature-displacement relations and the equations of equilibrium for $\rho = 0$ The former relations are given in Ref 1 by Eqs (29–30) and become, after simple substitutions,

$$\left.\begin{aligned} e_\xi &= u_\xi' + \omega_\xi w \\ e_\theta &= (1/\rho)\{nu_\theta + \rho' u_\xi + [1 - (\rho')^2]^{1/2} w\} \\ e_{\xi\theta} &= (-1/2\rho)[\rho' u_\theta + nu_\xi - \rho u_\theta'] \\ k_\xi &= [-w' + \omega_\xi u_\xi]' = \phi_\xi' \\ k_\theta &= (1/\rho)^2[\rho\rho'\phi_\xi + n^2 w + n\rho\omega_\theta u_\theta] \\ k_{\xi\theta} &= (1/2\rho)^2\{-n\rho\phi_\xi + n\rho w' - 2n\rho' w - \\ &\qquad 2\rho\rho'\omega_\theta u_\theta + \rho\rho'\omega_\xi u_\theta + \rho^2 w_\theta u'_\theta + \\ &\qquad \tfrac{1}{2}(\omega_\theta - \omega_\xi)\rho[nu_\xi + \rho' u_\theta + \rho u'_\theta]\} \end{aligned}\right\} \quad (2)$$

At the pole $r = 0$, the following relations hold:

$$\rho = 0 \qquad \rho' = 1 \qquad \rho'' = 0 \qquad \omega_\xi = \omega_\theta \quad (3)$$

From Eqs (2) and (3) one may see that, at the pole, the expressions for the strains e_θ and $e_{\xi\theta}$ have a first-order zero in the denominator, whereas the expressions for the curvatures k_θ and $k_{\xi\theta}$ have a second order zero in the denominator

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